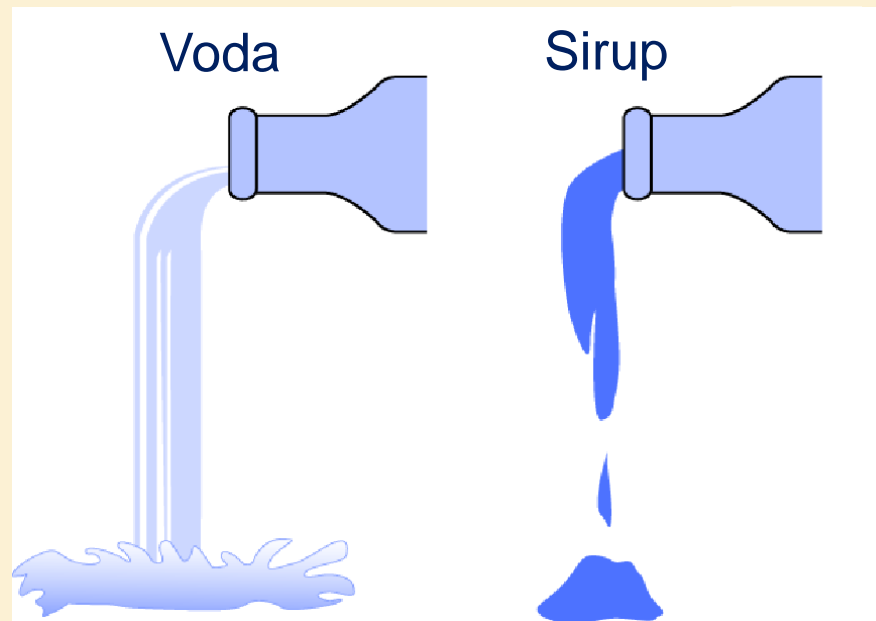


# **VISKOZNOST TEČNOSTI**

# Viskoznost tečnosti

---

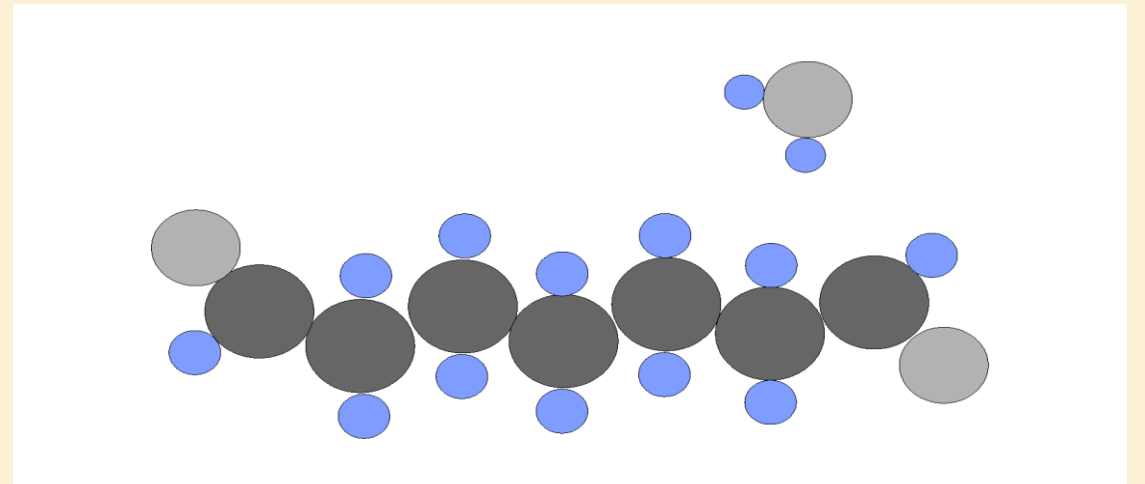
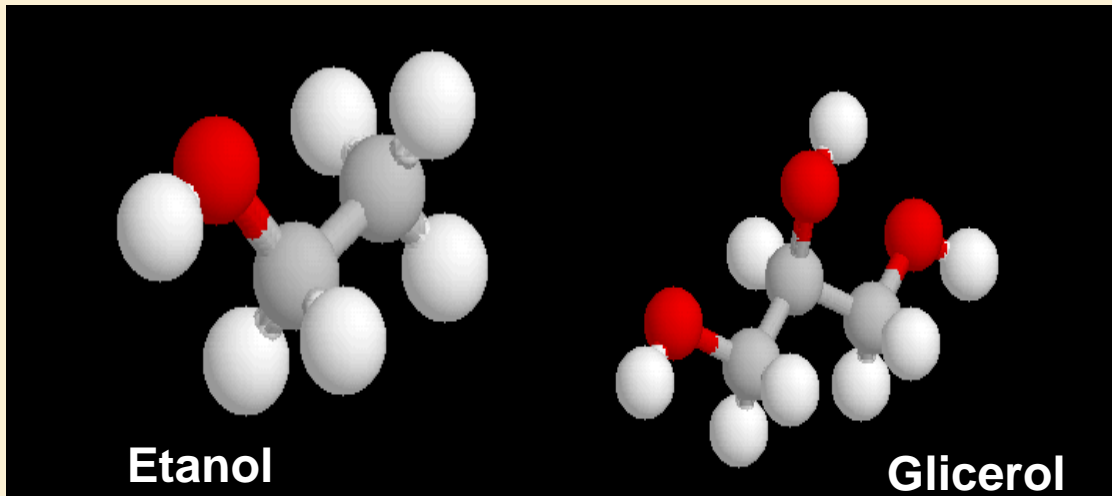
**Viskoznost:** otpor kojim se pojedini slojevi tečnosti suprotstavljaju kretanju jednih u odnosu na druge (vrsta **unutrašnjeg trenja** koja dovodi do protoka fluida konstantnom brzinom).



# Uzroci otpora proticanju tečnosti

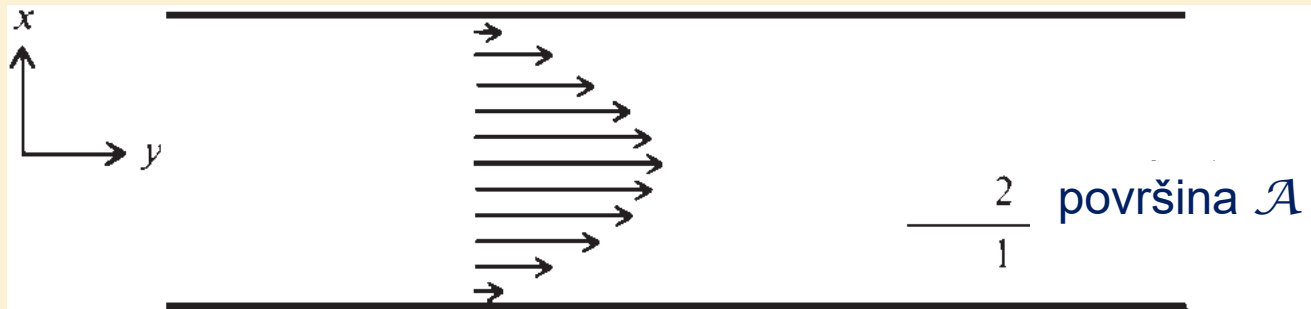
---

- međumolekulske interakcije
- oblik i veličina molekula



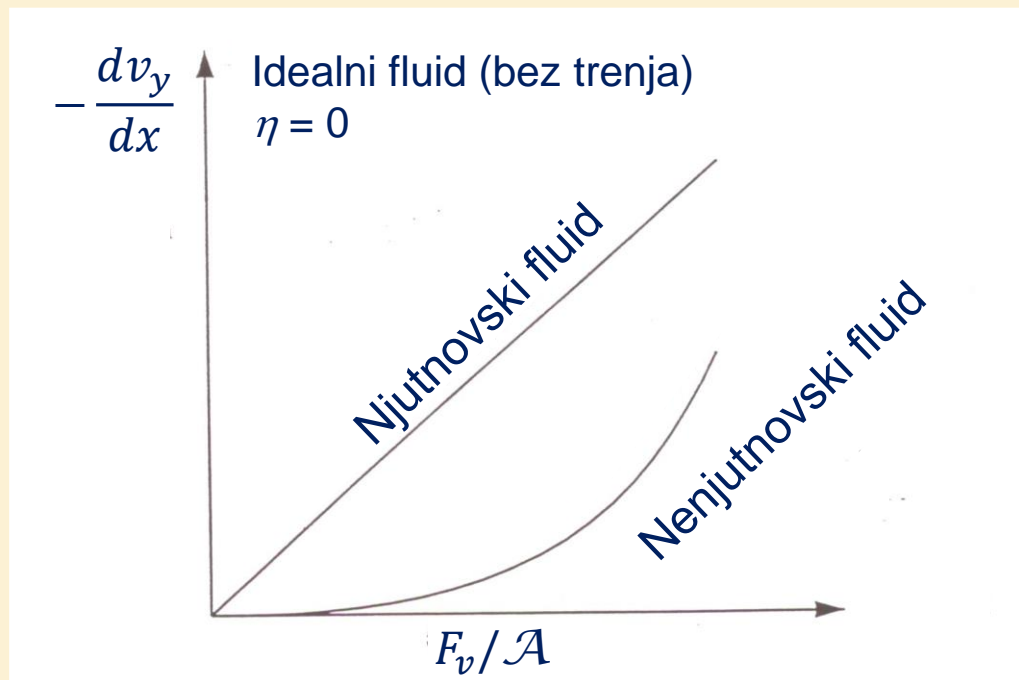
# Model

fluid protiče između dve paralelne ploče



Njutnov zakon viskoznosti:

$$F_v = -\eta \mathcal{A} \frac{dv_y}{dx}$$



$$\eta = -\frac{F_v / \mathcal{A}}{dv_y / dx}$$

$\eta$  : dinamički koeficijent viskoznosti  
jedinica je poaz: **1 P = 0,1 Pa s**

# Dinamički i kinematički koeficijent viskoznost

---

koeficijent viskoznosti	definicija	jedinica
dinamički	$\eta = - \frac{F_v / \mathcal{A}}{dv_y / dx}$	Poaz: 1 P = 0,1 Pa s
kinematički	$\nu = \frac{\eta}{\rho}$	Stoks: 1 St = 10 <sup>-4</sup> m <sup>2</sup> s <sup>-1</sup>

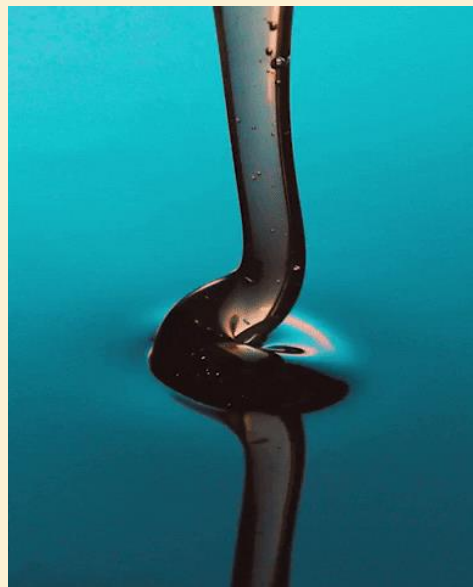
# Dinamički i kinematički koeficijent viskoznost

	temperatura / °C	dinam. koef. viskoznost / Pa s	gustina / kg/m <sup>3</sup>	kinem. koef. viskoznosti / m <sup>2</sup> /s
vazduh	0	$17,09 \cdot 10^{-6}$	1,293	$13,22 \cdot 10^{-6}$
	20	$18,08 \cdot 10^{-6}$	1,205	$15,00 \cdot 10^{-6}$
	40	$19,04 \cdot 10^{-6}$	1,128	$16,88 \cdot 10^{-6}$
sveža voda	0	$1,787 \cdot 10^{-3}$	$1,000 \cdot 10^3$	$1,787 \cdot 10^{-6}$
	20	$1,002 \cdot 10^{-3}$	$0,998 \cdot 10^3$	$1,004 \cdot 10^{-6}$
	40	$0,653 \cdot 10^{-3}$	$0,992 \cdot 10^3$	$0,658 \cdot 10^{-6}$
morska voda	20	$1,072 \cdot 10^{-3}$	$1,024 \cdot 10^3$	$1,047 \cdot 10^{-6}$
aceton	20	$0,326 \cdot 10^{-3}$	$0,792 \cdot 10^3$	$0,412 \cdot 10^{-6}$
glicerin	20	$1,490 \cdot 10^{-3}$	$1,261 \cdot 10^3$	$1,182 \cdot 10^{-3}$
živa	20	$1,554 \cdot 10^{-3}$	$13,546 \cdot 10^3$	$0,115 \cdot 10^{-6}$

# Fluidnost

---

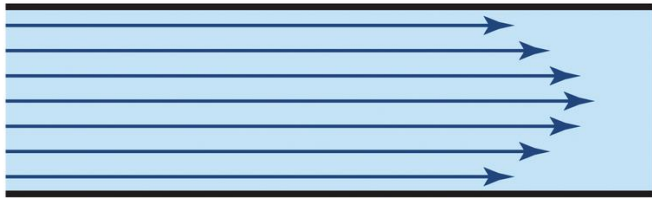
koeficijent fluidnosti:  $\phi = \frac{1}{\eta}$   
(pokazuje lakoću kojom tečnost teče)



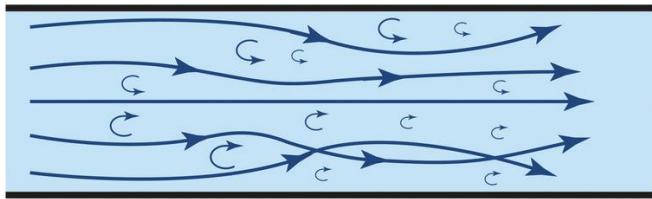
# Vrste protoka

---

laminarni protok



turbulentni protok



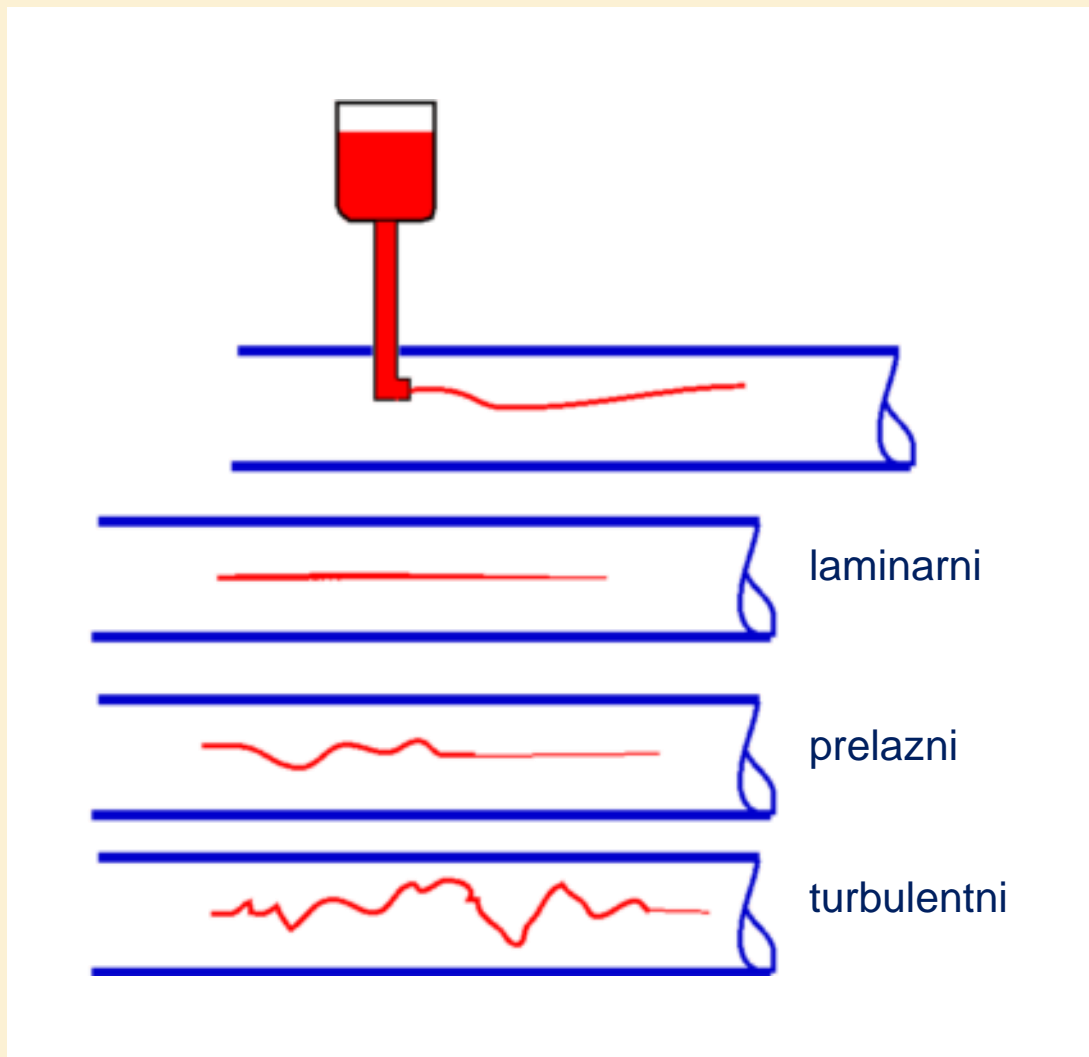
laminarni protok



turbulentni protok



# Rejnoldsov eksperiment



$$Re = \frac{\rho v d}{\eta}$$

$\rho$  – gustina fluida

$v$  – srednja brzina kretanja fluida kroz cev

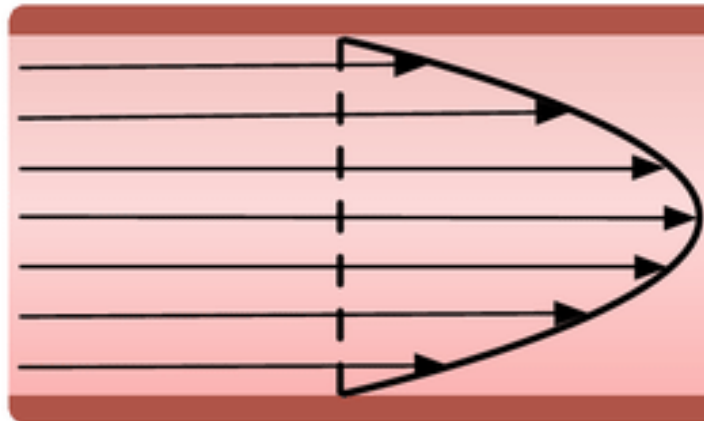
$d$  – prečnik cevi

$\eta$  – koeficijent viskoznosti fluida

# Tipovi protoka fluida

$$R_e = \frac{\rho v d}{\eta}$$

Laminarni protok ( $R_e < 2000$ )



Prelazni protok



Turbulentni protok ( $R_e > 4000$ )

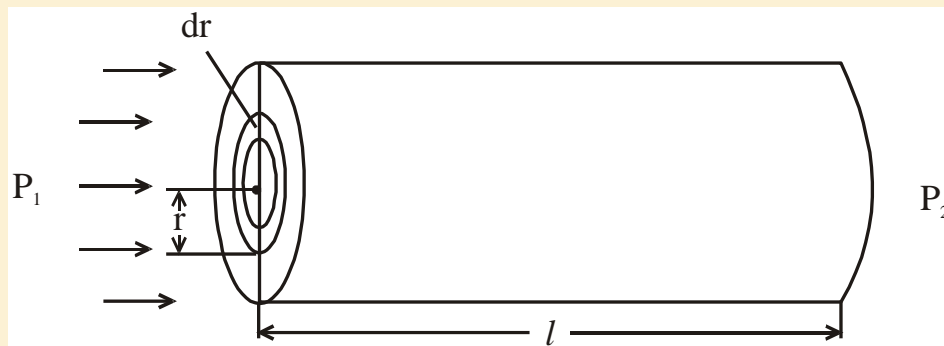


Uobičajeni tip protoka pri sporom proticanju.

Česti tip protoka pri brzom proticanju.

# Poazejev zakon

Posmatra se stacionarno proticanje nestišljivog fluida kroz cev pod dejstvom konstantne razlike pritiska.



$$F^r = \Delta P \cdot \pi r^2$$

$$F_v^r = -\eta \frac{dv}{dr} 2\pi r l$$

$$-\eta \frac{dv}{dr} \cdot 2\pi r l = \pi r^2 \Delta P$$

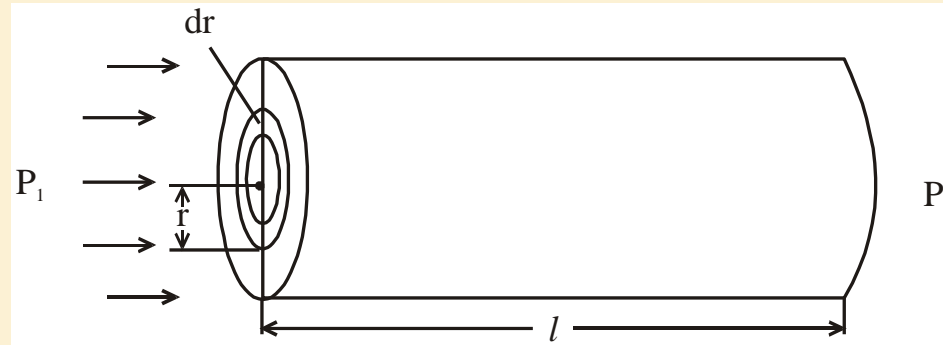
$$dv = -\frac{\Delta P}{2\eta l} r dr$$

$$\int_v^0 dv = -\frac{\Delta P}{2\eta l} \int_r^R r dr$$

$$v = \frac{\Delta P}{4\eta l} (R^2 - r^2)$$

# Poazejev zakon

---



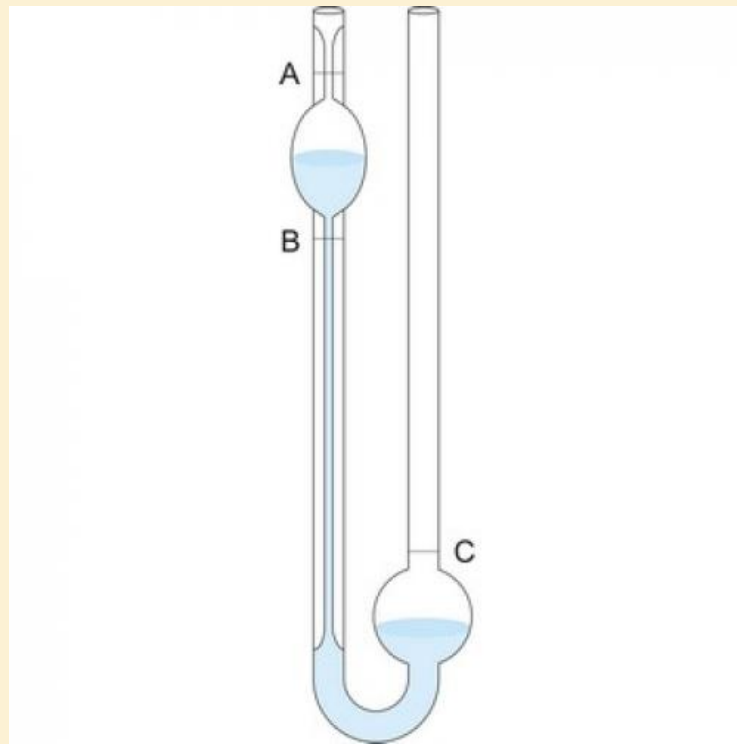
$$dV = vt2\pi r dr = \frac{\Delta P}{2\eta l} \pi (R^2 r - r^3) dr$$

$$V = \frac{\Delta P \pi t}{2\eta l} \int_0^R (R^2 r - r^3) dr$$

$$V = \frac{\Delta P \pi R^4 t}{8\eta l}$$

# Merenje koeficijenta viskoznosti: Ostvaldov viskozimetar

---



$$\eta = \frac{\Delta P \cdot \pi R^4 t}{8Vl} = \frac{\rho g h \cdot \pi R^4 t}{8Vl}$$

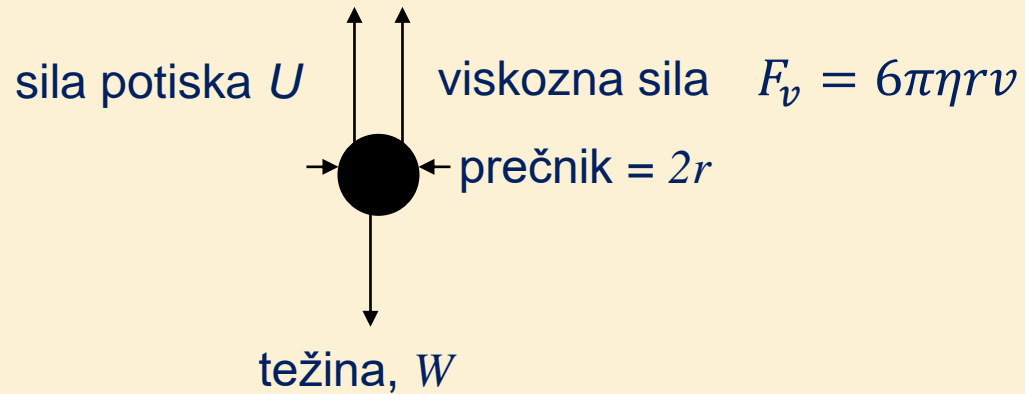
$$\frac{\eta}{\eta_0} = \frac{t}{t_0} \frac{\rho}{\rho_0}$$

# Stoksov zakon

---

$$U = m_l g = \frac{4}{3} \pi r^3 \rho_l g$$

$$W = m_s g = \frac{4}{3} \pi r^3 \rho_s g$$



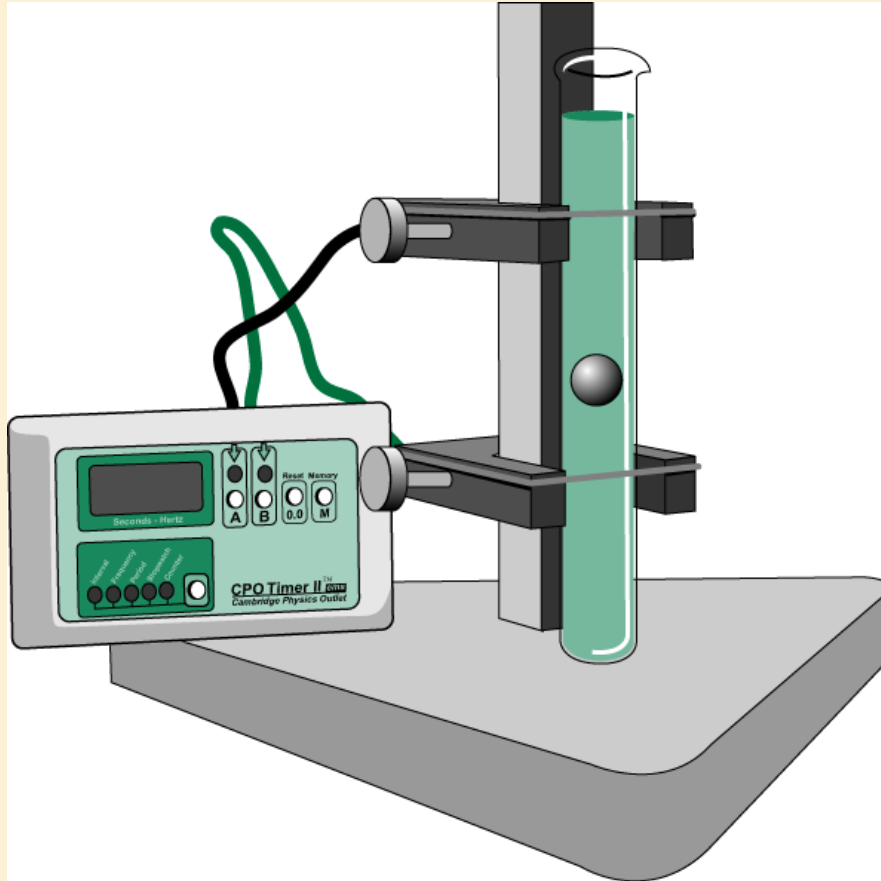
U stanju ravnoteže nema ubrzanja:  $U - W + F = 0$

$$\frac{4}{3} \pi r^3 (\rho_l - \rho_s) g + 6\pi\eta r \frac{l}{t} = 0$$

$$\boxed{\eta = \frac{2}{9l} r^2 g (\rho_s - \rho_l) t}$$

# Merenje koeficijenta viskoznosti: Stoksov zakon

---

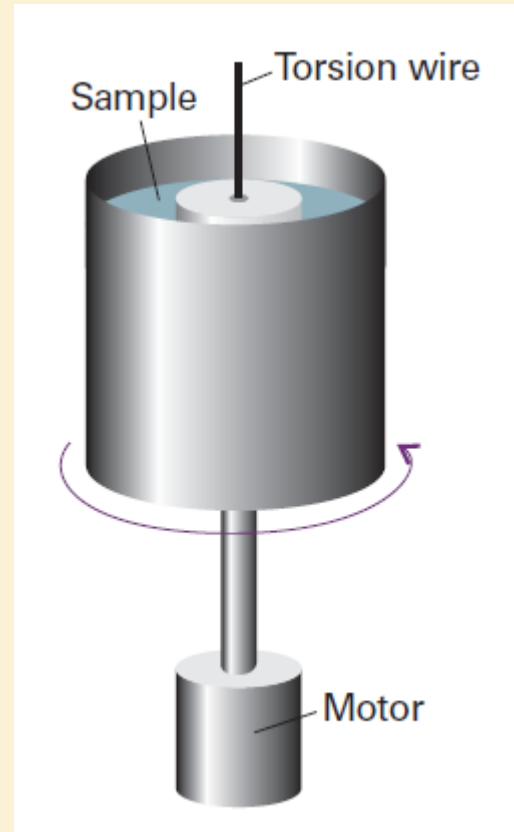


$$\eta = \frac{2t}{9l} r^2 g (\rho - \rho')$$

$$\frac{\eta_1}{\eta_2} = \frac{(\rho - \rho'_1)t_1}{(\rho - \rho'_2)t_2}$$

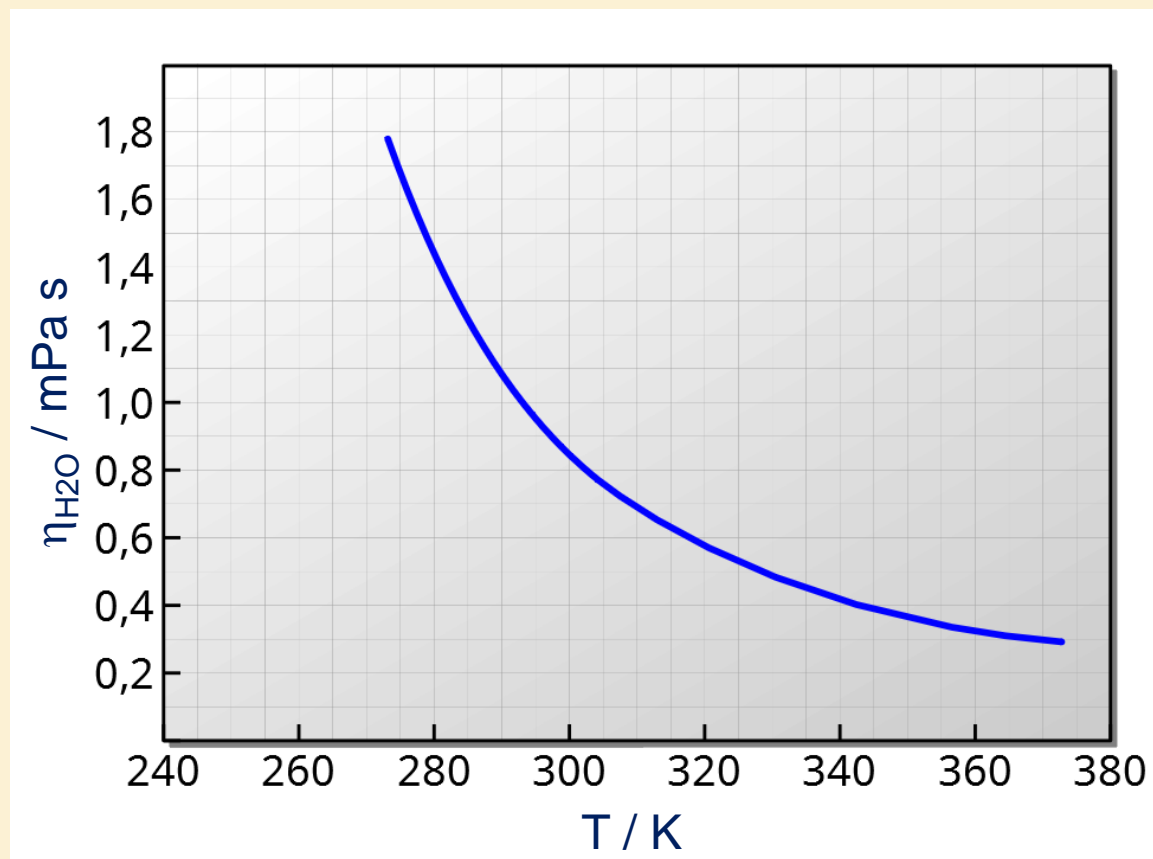
# Merenje koeficijenta viskoznosti: rotacioni viskozimetar

---





# Viskoznost i temperatura



Arenijus i Gucman:  $\eta = A \exp\left(\frac{B}{RT}\right)$

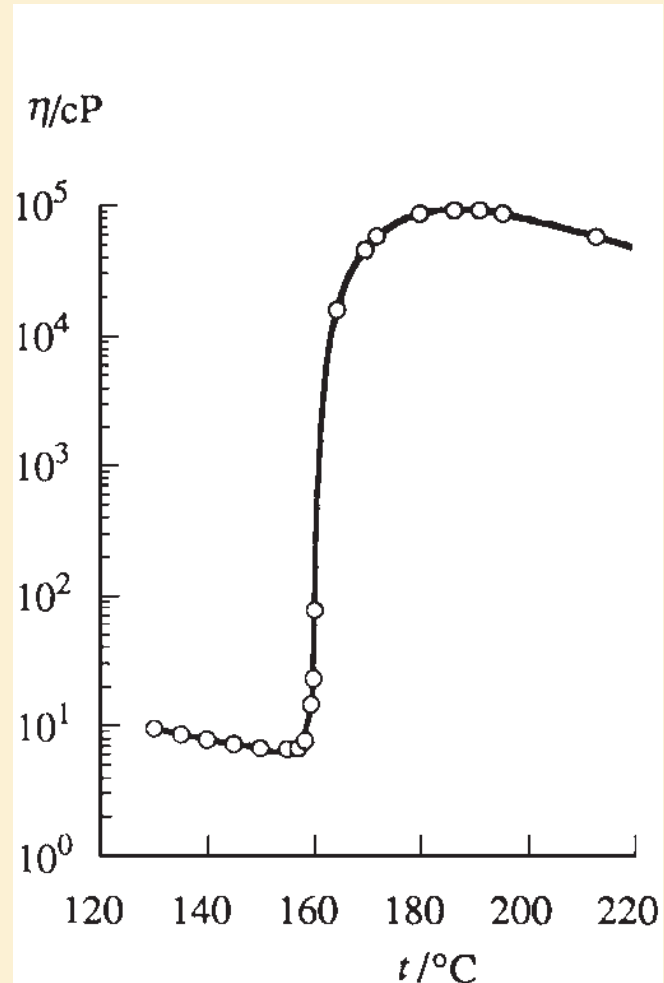
Andrade:  $\eta v_{sp}^{1/2} = A \exp\left(\frac{B}{RT v_{sp}}\right)$

Bačinski:  $\eta = \frac{c}{v_{sp} - b}$

$v_{sp} - b \approx$  zapremina šupljina

# Anomalous behavior of liquid sulfur

---

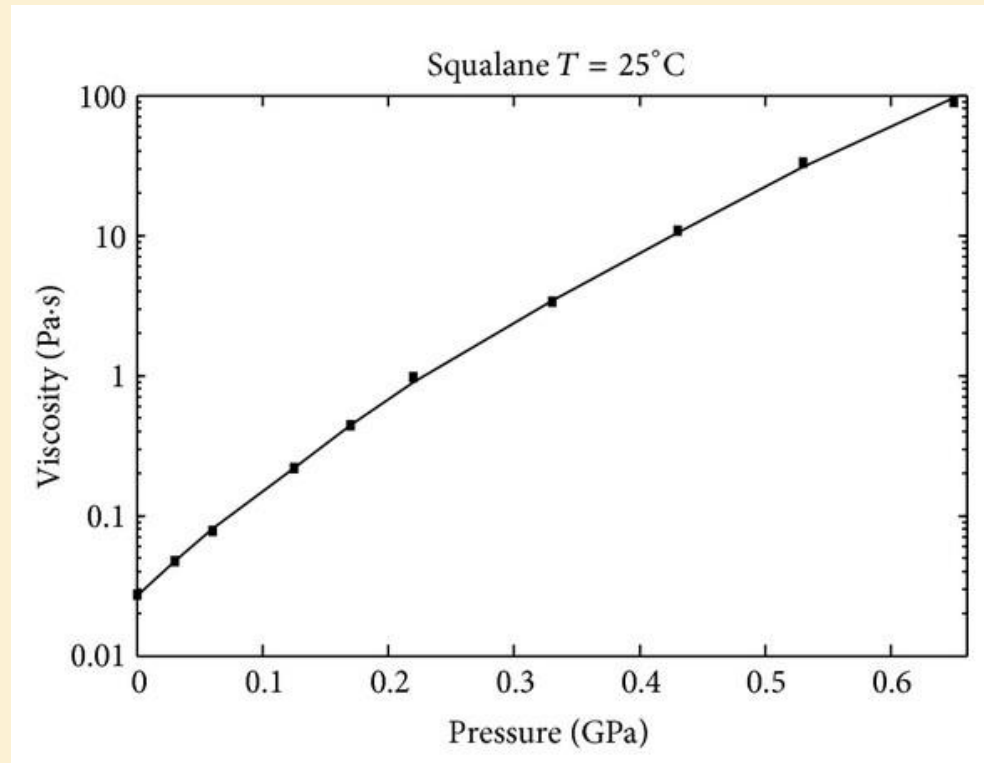


# Viskoznost i pritisak

---

$$\eta = \eta_0 \exp\left(\frac{P\Delta V_h}{RT}\right)$$

$\Delta V_h$  - zapremina šupljina po molu tečnosti



# Viskoznost binarnih sistema

---

Relativna viskoznost:  $\eta_r = \frac{\eta}{\eta_0}$

Specifična viskoznost:  $\eta_{sp} = \frac{\eta - \eta_0}{\eta_0} = \frac{\eta}{\eta_0} - 1 = \eta_r - 1$

Redukovana viskoznost:  $\eta_{red} = \frac{\eta_{sp}}{c}$

Unutrašnja viskoznost:  $[\eta] = \lim_{c \rightarrow 0} \frac{\eta_{sp}}{c}$

# Primer: polimeri

---

Mark-Houvinkova relacija:

$$[\eta] = K \bar{M}^a$$

$[\eta]$  - unutrašnja viskoznost

$K, a$  - konstante za posmatrani sistem polimer – tečnost

$\bar{M}$  - srednja molarna masa polimera