IDEALNI SISTEMI

→ PRIMENA KVANTNIH STATISTIKA NA IDEALNE GASOVE:

5.1) p^+, n, e^-, $\alpha = {}^4He^{2+}$, ${}^3He$, D, H, ${}^4He$

F F F B F F B B

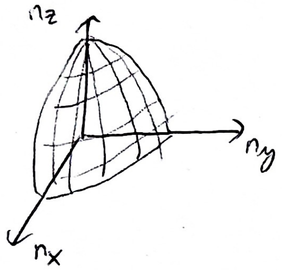
5.2) GUSTINA JEDNOČESTIČNIH STANJA $g(\varepsilon)$:

$$\vec{k} = \frac{\pi}{L}(n_x\vec{i} + n_y\vec{j} + n_z\vec{k}) \; ; \; n_x, n_y, n_z = 1,2,3,\ldots$$ → STOJEĆI TALAS

$V = L^3$

$|\vec{k}| \equiv k = \sqrt{\vec{k}\cdot\vec{k}}$

$$k^2 = \vec{k}\cdot\vec{k} = \frac{\pi^2}{L^2}(n_x^2+n_y^2+n_z^2) \Rightarrow n_x^2+n_y^2+n_z^2 = \left(\frac{Lk}{\pi}\right)^2$$ ⇒ JEDNAČINA SFERE



→ ZA DOVOLJNO VELIKE n_x, n_y, n_z OVO POSTAJE J-NA SFERE

$$\begin{pmatrix}\text{BROJ}\\ \text{STANJA}\end{pmatrix} = \frac{1}{8}\begin{pmatrix}\text{ZAPREMINE}\\ \text{SFERE}\end{pmatrix}$$

$$R = \frac{Lk}{\pi}$$

→ BROJ STANJA: $\Phi(k) = \frac{1}{8}\cdot\frac{4}{3}R^3\pi = \frac{1}{\cancel{8}_2}\cdot\frac{\cancel{4}}{3}\cancel{\pi}\cdot\frac{L^3k^3}{\pi^{\cancel{3}\,2}}$

$$\boxed{\Phi(k) = \frac{L^3k^3}{6\pi^2} = \frac{Vk^3}{6\pi^2}}$$ → UKUPAN BROJ STANJA SISTEMA

→ GUSTINA STANJA $\equiv \dfrac{\begin{pmatrix}\text{BROJ}\\ \text{STANJA}\end{pmatrix}}{\begin{pmatrix}\text{JEDINICA}\\ \text{ENERGIJE}\end{pmatrix}}$ $\Rightarrow g(k) = \dfrac{d\Phi(k)}{dk} = \dfrac{\cancel{3}Vk^2}{\cancel{6}_2\pi^2} = \dfrac{k^2V}{2\pi^2}$

↳ E ZAVISI OD k, PA UMESTO $\frac{d}{dE}$ MOŽEMO DA PIŠEMO $\frac{d}{dk}$

$g(k) \longrightarrow g(\varepsilon)$

$g(k)\,dk = g(\varepsilon)\,d\varepsilon$

$$g(\varepsilon) = g(k)\frac{dk}{d\varepsilon} = \frac{k^2V}{2\pi^2}\frac{dk}{d\varepsilon}$$

→ RAZLIČITI SLUČAJEVI:

1) FERMIONI I BOZONI SA MASOM

$$\varepsilon = \frac{p^2}{2m} \qquad h = p\lambda \Rightarrow p = \frac{h}{\lambda}\cdot\frac{2\pi}{2\pi} = \hbar k$$

$$\varepsilon = \frac{\hbar^2k^2}{2m} \Rightarrow k = \frac{\sqrt{2m\varepsilon}}{\hbar} \;\Big/\frac{d}{d\varepsilon} \Rightarrow \frac{dk}{d\varepsilon} = \frac{1}{\hbar}\cdot\frac{1}{2\sqrt{2m\varepsilon}}\cdot 2m = \frac{1}{2\hbar}\sqrt{\frac{4m^2}{2m\varepsilon}} = \frac{1}{2\hbar}\sqrt{\frac{2m}{\varepsilon}}$$

$$g(\varepsilon) = \frac{k^2V}{2\pi^2}\frac{dk}{d\varepsilon} = \frac{V}{2\pi^2}\cdot\frac{2m\varepsilon}{\hbar^2}\cdot\frac{1}{2\hbar}\sqrt{\frac{2m}{\varepsilon}} = \frac{1}{4}\cdot V\cdot\pi^{-2}\cdot 2m\cdot\varepsilon\cdot\hbar^{-2}\cdot\hbar^{-1}\cdot(2m)^{\frac{1}{2}}\cdot\varepsilon^{-\frac{1}{2}} =$$

$$= \frac{1}{4}\cdot V\cdot\pi^{-2}\cdot(2m)^{\frac{3}{2}}\cdot\varepsilon^{\frac{1}{2}}\cdot\hbar^{-3} \qquad \hbar = \frac{h}{2\pi}$$

$$g(\mathcal{E}) = \frac{1}{4}\cdot V\cdot \pi^{-2}\cdot (2m)^{\frac{3}{2}}\cdot\sqrt{\mathcal{E}}\cdot h^{-3}\cdot(2\pi)^3 = \frac{1}{4}\cdot 8V\cdot\pi\cdot\sqrt{\mathcal{E}}\cdot\left(\frac{2m}{h^2}\right)^{\frac{3}{2}} = 2\pi V\left(\frac{2m}{h^2}\right)^{\frac{3}{2}}\cdot\sqrt{\mathcal{E}}$$

$$\boxed{g(\mathcal{E}) = 2\pi V\left(\frac{2m}{h^2}\right)^{\frac{3}{2}}\cdot\sqrt{\mathcal{E}}}$$

→ GUSTINA STANJA ZA BOZONE I FERMIONE SA MASOM

$g(\mathcal{E}) \sim \mathcal{E}^{\frac{1}{2}}$

2) BOZONI BEZ MASE:

$$\mathcal{E} = h\nu = h\cdot\frac{c}{\lambda}\cdot\frac{2\pi}{2\pi} = \hbar k c \Rightarrow k = \frac{\mathcal{E}}{\hbar\cdot c} \Big/ \frac{d}{d\mathcal{E}} \Rightarrow \frac{dk}{d\mathcal{E}} = \frac{1}{\hbar c}$$

$$g(\mathcal{E}) = \frac{k^2 V}{2\pi^2}\cdot\frac{dk}{d\mathcal{E}} = \frac{\mathcal{E}^2 V}{\hbar^2 c^2\cdot 2\pi^2}\cdot\frac{1}{\hbar c} = \frac{\mathcal{E}^2 V}{\hbar^3\cdot c^3\cdot 2\pi^2} = \frac{\mathcal{E}^2 V}{\frac{h^3}{\cancel{8}\pi^{\cancel{3}}}\cdot c^3\cdot \cancel{2\pi^2}} = \frac{4\pi\mathcal{E}^2 V}{(hc)^3}$$

$$\boxed{g(\mathcal{E}) = 4\pi V\cdot\frac{1}{(hc)^3}\cdot\mathcal{E}^2}$$

→ GUSTINA STANJA ZA BOZONE BEZ MASE

$g(\mathcal{E}) \sim \mathcal{E}^2$

→ IDEALNI FOTONSKI GAS:

5.3 IDEALNI FOTONSKI GAS ≡ ELEKTROMAGNETNO POLJE $\langle N\rangle, \langle E\rangle, p = ?$

→ GUSTINA STANJA: $g(\mathcal{E}) = 2\cdot\left[4\pi V\cdot\frac{1}{(hc)^3}\mathcal{E}^2\right]$

↳ ZBOG DVOSTRUKE POLARIZACIJE ELEKTRIČNOG I MAGNETNOG POLJA

$$\Xi_{FD/BA} = \prod_k \left(1 \pm \lambda e^{-\beta\mathcal{E}_k}\right)^{\pm 1} \xrightarrow{\text{BOZONI}} \Xi_{BA} = \prod_k\left(1-\lambda e^{-\beta\mathcal{E}_k}\right)^{-1}$$

$$\Xi = \prod_k \frac{1}{1-\lambda e^{-\beta\mathcal{E}_k}} \qquad \langle n_k\rangle = \frac{1}{e^{\beta(\mathcal{E}_k-\mu)}-1} = \frac{\lambda e^{-\beta\mathcal{E}_k}}{1-\lambda e^{-\beta\mathcal{E}_k}}$$

SREDNJA ZAUZETOST JEDNOČESTIČNOG STANJA

$$\hookrightarrow \frac{1}{\lambda e^{-\beta\mathcal{E}_k}} = \frac{1}{\lambda}e^{\beta\mathcal{E}_k} = \frac{1}{e^{\beta\mu}}\cdot e^{\beta\mathcal{E}_k} = e^{\beta(\mathcal{E}_k-\mu)}$$

$$\langle N\rangle = \sum_k\langle n_k\rangle = \sum_k\frac{\lambda e^{-\beta\mathcal{E}_k}}{1-\lambda e^{-\beta\mathcal{E}_k}} \quad (1)$$

$$\langle E\rangle = \sum_k \mathcal{E}_k\langle n_k\rangle = \sum_k\frac{\mathcal{E}_k\lambda e^{-\beta\mathcal{E}_k}}{1-\lambda e^{-\beta\mathcal{E}_k}} \quad (2)$$

$$\Omega = -pV = -kT\ln\Xi = -kT\sum_k \ln\left(1-\lambda e^{-\beta\mathcal{E}_k}\right)^{-1} = kT\sum_k\ln\left(1-\lambda e^{-\beta\mathcal{E}_k}\right)$$

$$p = -\frac{kT}{V}\sum_k\ln\left(1-\lambda e^{-\beta\mathcal{E}_k}\right) \quad (3)$$

$$\boxed{\sum_k f(\mathcal{E}_k) \longrightarrow \int \underbrace{g(\mathcal{E})}_{\text{gustina stanja}} f(\mathcal{E})\,d\mathcal{E}}$$

→ OVAKO ĆEMO OVE 3 SUME DA PREBACIMO U INTEGRALE

→ DA BISMO LAKŠE REŠAVALI INTEGRALE, E-JU KOJA FIGURIŠE U IZRAZIMA ĆEMO ZAPISATI KAO $\mathcal{E} = \omega\hbar$

$g(\mathcal{E}) \longrightarrow g(\omega)$

$g(\mathcal{E})\,d\mathcal{E} = g(\omega)\,d\omega$

$g(\omega) = g(\mathcal{E})\frac{d\mathcal{E}}{d\omega} \longrightarrow \mathcal{E} = \omega\hbar, \quad \frac{d\mathcal{E}}{d\omega} = \hbar$

$$g(\omega) = 8\pi V\frac{1}{(hc)^3}\mathcal{E}^2\cdot\hbar = \frac{8\pi V}{(hc)^3}\cdot(\omega\hbar)^2\cdot\hbar \qquad h = 2\pi\hbar$$

$$g(\omega) = \frac{\cancel{8}\cancel{\pi}V}{\cancel{8}\pi^{\cancel{3}}\cancel{\hbar^3}\cdot c^3}\cdot\omega^2\cdot\cancel{\hbar^3} \qquad \boxed{g(\omega) = \frac{V\cdot\omega^2}{\pi^2 c^3}}$$

$$g(\omega) = \frac{V\omega^2}{\pi^2 c^3} \qquad \lambda = 1 \qquad \varepsilon = \omega\hbar$$

$$(1) \rightarrow \langle N \rangle = \sum_k \frac{e^{-\beta\varepsilon_k}}{1-e^{-\beta\varepsilon_k}} = \int_0^\infty g(\omega)\cdot\frac{e^{-\beta\omega\hbar}}{1-e^{-\beta\omega\hbar}}\,d\omega = \frac{V}{\pi^2c^3}\int_0^\infty \frac{\omega^2\cdot e^{-\beta\omega\hbar}}{1-e^{-\beta\omega\hbar}}\,d\omega \quad (4)$$

$$(2) \rightarrow \langle E \rangle = \sum_k \frac{\varepsilon_k\cdot e^{-\beta\varepsilon_k}}{1-e^{-\beta\varepsilon_k}} = \int_0^\infty g(\omega)\cdot\frac{\omega\hbar\cdot e^{-\beta\omega\hbar}}{1-e^{-\beta\omega\hbar}}\,d\omega = \frac{V\hbar}{\pi^2c^3}\int_0^\infty \frac{\omega^3\cdot e^{-\beta\omega\hbar}}{1-e^{-\beta\omega\hbar}}\,d\omega \quad (5)$$

$$(3) \rightarrow p = -\frac{kT}{V}\sum_k \ln(1-e^{-\beta\varepsilon_k}) = -\frac{kT}{V}\cdot\int_0^\infty g(\omega)\cdot\ln(1-e^{-\beta\omega\hbar})\,d\omega =$$

$$= -\frac{kT}{\cancel{V}}\cdot\frac{\cancel{V}}{\pi^2c^3}\cdot\int_0^\infty \omega^2\cdot\ln(1-e^{-\beta\omega\hbar})\,d\omega = -\frac{kT}{\pi^2c^3}\int_0^\infty \omega^2\ln(1-e^{-\beta\omega\hbar})\,d\omega \quad (6)$$

→ ZA INTEGRALE (4) i (5) KORISTIMO SLEDEĆI TRIK:

$$\frac{e^{-\beta\omega\hbar}}{1-e^{-\beta\omega\hbar}} = e^{-\beta\omega\hbar}\cdot\underbrace{\frac{1}{1-e^{-\beta\omega\hbar}}}_{\text{STEPENI RED}} = e^{-\beta\omega\hbar}\cdot\sum_{n=0}^\infty (e^{-\beta\omega\hbar})^n = \sum_{n=0}^\infty (e^{-\beta\omega\hbar})^{n+1} =$$

$$= \sum_{n=1}^\infty e^{-\beta n\omega\hbar}$$

→ ZA INTEGRAL (6):

$$\ln(1-e^{-\beta\omega\hbar}) = -\sum_{n=1}^\infty \frac{e^{-\beta\omega\hbar n}}{n} = -\sum_{n=1}^\infty \frac{(e^{-\beta\omega\hbar})^n}{n}$$

$$(4) \rightarrow \langle N \rangle = \frac{V}{\pi^2c^3}\int_0^\infty \omega^2\cdot\sum_{n=1}^\infty e^{-\beta n\omega\hbar}\,d\omega = \left\{ \begin{array}{l} \text{SMENA:} \\ \beta n\omega\hbar = x \Rightarrow \omega = \frac{x}{\beta n\hbar} \quad d\omega = \frac{1}{\beta n\hbar}dx \end{array}\right.$$

$$= \frac{V}{\pi^2c^3}\int_0^\infty\sum_{n=1}^\infty \frac{x^2}{(\beta n\hbar)^2}\cdot e^{-x}\cdot\frac{1}{\beta n\hbar}\,dx = \frac{V}{\pi^2c^3}\cdot\frac{1}{(\beta\hbar)^3}\cdot\sum_{n=1}^\infty\frac{1}{n^3}\underbrace{\int_0^\infty x^2e^{-x}dx}_{\Gamma(3)=2!=2}$$

$$\boxed{\langle N \rangle = \frac{2V(kT)^3}{\pi^2(c\hbar)^3}\sum_{n=1}^\infty\frac{1}{n^3}}$$

$$(5) \rightarrow \langle E \rangle = \frac{V\hbar}{\pi^2c^3}\int_0^\infty \omega^3\sum_{n=1}^\infty e^{-\beta n\omega\hbar}\,d\omega = \left\{ \begin{array}{l} \text{SMENA:} \\ \beta n\omega\hbar = x \Rightarrow \omega = \frac{x}{\beta n\hbar} \quad d\omega = \frac{1}{\beta n\hbar}dx \end{array}\right.$$

$$= \frac{V\hbar}{\pi^2c^3}\int_0^\infty\sum_{n=1}^\infty \frac{x^3}{(\beta n\hbar)^3}\cdot e^{-x}\frac{1}{\beta n\hbar}\,dx = \frac{V\hbar}{\pi^2c^3}\cdot\frac{1}{(\beta\hbar)^4}\sum_{n=1}^\infty\frac{1}{n^4}\underbrace{\int_0^\infty x^3\cdot e^{-x}dx}_{\Gamma(4)=3!=6} =$$

$$= \frac{6V\hbar}{\pi^2c^3}\cdot\frac{(kT)^4}{\hbar^4}\sum_{n=1}^\infty\frac{1}{n^4} \Rightarrow \boxed{\langle E \rangle = \frac{6V(kT)^3}{\pi^2(c\hbar)^3}\cdot kT\sum_{n=1}^\infty\frac{1}{n^4}}$$

$$(6) \rightarrow p = -\frac{kT}{\pi^2c^3}\int_0^\infty \omega^2\left(-\sum_{n=1}^\infty\frac{e^{-\beta n\omega\hbar}}{n}\right)d\omega =$$

$$= +\frac{kT}{\pi^2c^3}\int_0^\infty\sum_{n=1}^\infty\frac{1}{n}\omega^2e^{-\beta n\omega\hbar}\,d\omega = \left\{ \begin{array}{l} \text{SMENA:} \\ \beta n\omega\hbar = x \Rightarrow \omega = \frac{x}{\beta n\hbar} \quad d\omega = \frac{dx}{\beta n\hbar} \end{array}\right.$$

$$p = \frac{kT}{\pi^2 c^3} \int_0^\infty \sum_{n=1}^\infty \frac{1}{n} \frac{x^2}{(\beta n \hbar)^2} e^{-x} \frac{dx}{\beta n \hbar} = \frac{kT}{\pi^2 c^3} \sum_{n=1}^\infty \frac{1}{n} \int_0^\infty \frac{x^2}{(\beta n\hbar)^2} \cdot e^{-x} \frac{dx}{\beta n \hbar} =$$

$$= \frac{kT}{\pi^2 c^3} \frac{1}{(\beta\hbar)^3} \sum_{n=1}^\infty \frac{1}{n^4} \underbrace{\int_0^\infty x^2 e^{-x} dx}_{\Gamma(3)=2!=2}$$

$$\boxed{p = \frac{2(kT)^3}{\pi^2 (c\hbar)^3} kT \sum_{n=1}^\infty \frac{1}{n^4}}$$

→ RIMANOVA ZETA F-JA:

$$\boxed{\zeta(x) = \sum_{n=1}^\infty n^{-x}} \quad \text{ili} \quad \zeta(x) = \frac{1}{\Gamma(x)} \cdot \int_0^\infty \frac{t^{x-1}}{e^x - 1} dt$$

GAMA F-JA

$\zeta(1) \to \infty$ $\qquad \zeta(2) = \frac{\pi^2}{6}$

$\zeta(3) = 1,20205\ldots$ $\qquad \zeta(4) = \frac{\pi^4}{90}$

$\zeta(5) = 1,03692\ldots$ $\qquad \zeta(6) = \frac{\pi^6}{945}$

$\vdots \qquad \vdots$

5.4 $g(\varepsilon) = A\varepsilon^B$; $A, B = const$ → GUSTINA JEDNOČESTIČNIH STANJA U OPŠTEM SLUČAJU

$pV = f(A,B)\langle E\rangle$

$f(A,B) = ?$

$$\langle E\rangle = \sum_i \langle n_i\rangle \varepsilon_i = \sum_i \frac{\varepsilon_i \lambda e^{-\beta\varepsilon_i}}{1\pm\lambda e^{-\beta\varepsilon_i}} = \int_0^\infty A\cdot\varepsilon^B \cdot \frac{\varepsilon\lambda e^{-\beta\varepsilon}}{1\pm\lambda e^{-\beta\varepsilon}} d\varepsilon = A\int_0^\infty \frac{\varepsilon^{B+1}\cdot\lambda e^{-\beta\varepsilon}}{1\pm\lambda e^{-\beta\varepsilon}} d\varepsilon$$

→ PARCIJALNA INTEGRACIJA: $I = u\cdot v - \int v\,du$

$u = \varepsilon^{B+1}$ $\qquad dv = \frac{\lambda e^{-\beta\varepsilon}d\varepsilon}{1\pm\lambda e^{-\beta\varepsilon}}$

$du = (B+1)\varepsilon^B$

$$v = \int \frac{\lambda e^{-\beta\varepsilon}}{1\pm\lambda e^{-\beta\varepsilon}} d\varepsilon = \Big\{ \text{SMENA: } 1\pm\lambda e^{-\beta\varepsilon} = t \Rightarrow \pm\lambda e^{-\beta\varepsilon}(-\beta)d\varepsilon = dt \Rightarrow \lambda e^{-\beta\varepsilon}d\varepsilon = \mp\frac{dt}{\beta}$$

$$v = \int \frac{\mp \frac{dt}{\beta}}{t} = \mp\frac{1}{\beta}\int\frac{dt}{t} = \mp kT\ln(t) = \mp kT \ln(1\pm\lambda e^{-\beta\varepsilon})$$

$$\langle E\rangle = A\cdot\Big[\mp kT\, \varepsilon^{B+1} \underbrace{\ln(1\pm\lambda e^{-\beta\varepsilon})\Big|_0^\infty}_{=0} \pm kT(B+1)\int_0^\infty \varepsilon^B \ln(1\pm\lambda e^{-\beta\varepsilon}) d\varepsilon\Big]$$

LOPITALOVO PRAVILO (POKAZATI)

$$\langle E\rangle = \pm kTA(B+1)\int_0^\infty \varepsilon^B\ln(1\pm\lambda e^{-\beta\varepsilon})d\varepsilon \quad (1)$$

$\Omega = -pV = -kT\ln\Xi$ $\qquad pV = \pm kT\sum_i \ln(1\pm e^{-\beta\varepsilon_i}) = \pm kT\int A\cdot\varepsilon^B\ln(1\pm e^{-\beta\varepsilon})d\varepsilon \quad (2)$

$\Xi = \prod_i (1\pm e^{-\beta\varepsilon_i})^{\pm1}$

$(2)\to(1) \Rightarrow \langle E\rangle = (B+1)pV \Rightarrow pV = \frac{1}{B+1}\langle E\rangle \Rightarrow \boxed{f(A,B) = \frac{1}{B+1}}$

$$f(A,B) = \frac{1}{B+1}$$

1) ZA BOZONE I FERMIONE SA MASOM:

$$g(\varepsilon) = 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sqrt{\varepsilon} \qquad g(\varepsilon) \sim \varepsilon^{\frac{1}{2}}$$

$$g(\varepsilon) = A \cdot \varepsilon^{B} \Rightarrow B = \frac{1}{2} \Rightarrow f(A,B) = \frac{1}{\frac{1}{2}+1} = \frac{2}{3} \Rightarrow \boxed{pV = \frac{2}{3}\langle E\rangle}$$

2) Bozoni BEZ MASE:

$$g(\varepsilon) = 4\pi V \frac{1}{(hc)^3} \varepsilon^2 \qquad g(\varepsilon) \sim \varepsilon^2$$

$$g(\varepsilon) = A \cdot \varepsilon^{B} \Rightarrow B = 2 \Rightarrow f(A,B) = \frac{1}{2+1} = \frac{1}{3} \Rightarrow \boxed{pV = \frac{1}{3}\langle E\rangle}$$

5.5 $S = k \sum_i \left[-\langle n_i\rangle \ln\langle n_i\rangle \mp (1 \mp \langle n_i\rangle) \ln(1 \mp \langle n_i\rangle) \right]$

$$\langle n_i\rangle = \frac{1}{e^{\beta(\varepsilon_i - \mu)} \pm 1} \quad (1) \qquad \Xi = \sum_k \sum_{N=0}^{\infty} e^{-\beta(E_k - \mu N)} \quad \text{Tj.} \quad \Xi = \prod_i \left(1 \pm \lambda e^{-\beta\varepsilon_i}\right)^{\pm 1}$$

$$\Omega = -kT \ln \Xi = \mp kT \sum_i \ln\left(1 \pm \lambda e^{-\beta\varepsilon_i}\right) = \mp kT \sum_i \ln\left(1 \pm e^{\beta(\mu - \varepsilon_i)}\right) \quad (2)$$

$$\text{Iz } (1) \Rightarrow e^{\beta(\varepsilon_i - \mu)} \pm 1 = \frac{1}{\langle n_i\rangle} \Rightarrow e^{\beta(\varepsilon_i-\mu)} = \frac{1}{\langle n_i\rangle} \mp 1 = \frac{1 \mp \langle n_i\rangle}{\langle n_i\rangle}$$

$$e^{\beta(\mu - \varepsilon_i)} = \frac{\langle n_i\rangle}{1 \mp \langle n_i\rangle} \quad (3)$$

$$(3) \to (2) \Rightarrow \Omega = \mp kT \sum_i \ln\left(1 \pm \frac{\langle n_i\rangle}{1 \mp \langle n_i\rangle}\right) =$$

$$= \mp kT \sum_i \ln\left(\frac{1 \mp \langle n_i\rangle \pm \langle n_i\rangle}{1 \mp \langle n_i\rangle}\right) = \mp kT \sum_i \ln\left(\frac{1 \mp \langle n_i\rangle}{1 \mp \langle n_i\rangle \pm \langle n_i\rangle}\right)^{-1} =$$

$$= \pm kT \sum_i \ln\left(\frac{1 \mp \langle n_i\rangle}{1 \mp \langle n_i\rangle \pm \langle n_i\rangle}\right) \Rightarrow \boxed{\Omega = \pm kT \sum_i \ln(1 \mp \langle n_i\rangle)} \quad (4)$$

$$\Omega = U - TS - \mu N \Rightarrow TS = \langle E\rangle - \mu\langle N\rangle - \Omega \Rightarrow S = \frac{\langle E\rangle - \mu\langle N\rangle - \Omega}{T}$$

$$\langle E\rangle = \sum_i \langle n_i\rangle \varepsilon_i \qquad \langle N\rangle = \sum_i \langle n_i\rangle$$

$$S = \frac{1}{T}\sum_i \langle n_i\rangle\varepsilon_i - \frac{\mu}{T}\sum_i \langle n_i\rangle \mp k \sum_i \ln(1 \mp \langle n_i\rangle) = k \sum_i \left[\frac{\varepsilon_i - \mu}{kT}\langle n_i\rangle \mp \ln(1 \mp \langle n_i\rangle)\right]$$

$$\frac{\varepsilon_i - \mu}{kT} = \beta(\varepsilon_i - \mu)$$

$$\frac{\varepsilon_i-\mu}{kT}=\beta(\varepsilon_i-\mu) \qquad (3)\ e^{\beta(\varepsilon_i-\mu)}=\frac{1\mp\langle n_i\rangle}{\langle n_i\rangle}$$

$$\beta(\varepsilon_i-\mu)=\ln\left(\frac{1\mp\langle n_i\rangle}{\langle n_i\rangle}\right)=\ln(1\mp\langle n_i\rangle)-\ln\langle n_i\rangle$$

$$S=k\cdot\sum_i\left[\frac{\varepsilon_i-\mu}{kT}\langle n_i\rangle\mp\ln(1\mp\langle n_i\rangle)\right]=$$

$$=k\sum_i\left[\left(\ln(1\mp\langle n_i\rangle)-\ln\langle n_i\rangle\right)\langle n_i\rangle\mp\ln(1\mp\langle n_i\rangle)\right]=$$

$$=k\sum_i\left[\langle n_i\rangle\ln(1\mp\langle n_i\rangle)-\langle n_i\rangle\ln\langle n_i\rangle\mp\ln(1\mp\langle n_i\rangle)\right]$$

$$\boxed{S=k\sum_i\left[-\langle n_i\rangle\ln\langle n_i\rangle\mp(1\mp\langle n_i\rangle)\ln(1\mp\langle n_i\rangle)\right]}$$

5.6 SLABODEGENERISANI GAS => MALA AKTIVNOST

$$\lambda=e^{\beta\mu}\approx 1 \Rightarrow \mu\approx 0$$

→ TADA VAŽI: $\langle E\rangle=A+B\rho+C\rho^2+\ldots$ $\qquad \rho=\frac{\langle N\rangle}{V}$

$p=A'+B'\rho+C'\rho+\ldots$

↳ NAJNIŽE KOREKCIJE

→ U OPŠTEM SLUČAJU ZNAMO DA VAŽI:

$$\langle N\rangle=\int_0^\infty g(\varepsilon)\cdot\frac{\lambda e^{-\beta\varepsilon}}{1\pm\lambda e^{-\beta\varepsilon}}d\varepsilon \quad (1)$$

$$\langle E\rangle=\int_0^\infty g(\varepsilon)\frac{\varepsilon\lambda e^{-\beta\varepsilon}}{1\pm\lambda e^{-\beta\varepsilon}}d\varepsilon \quad (2)$$

$$p=\pm\frac{kT}{V}\int_0^\infty g(\varepsilon)\ln(1\pm\lambda e^{-\beta\varepsilon})\,d\varepsilon \quad (3)$$

→ Razvoji koje ćemo koristiti:

$$\boxed{\frac{1}{1\pm x}=\sum_{n=0}^{\infty}(\mp1)^n x^n=\sum_{n=1}^{\infty}(\mp1)^{n-1}x^{n-1}}$$

$$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$$

$$\ln(1-x)=\sum_{n=1}^{\infty}(-1)^{2n+1}\frac{x^n}{n}$$

Tj. $\boxed{\ln(1\pm x)=\sum_{n=1}^{\infty}(-1)^{\substack{n+1\\2n+1}}\frac{x^n}{n}}$

→ Slučaj bozona i fermiona sa masom:

$$g(\varepsilon)=2\pi V\left(\frac{2m}{h^2}\right)^{\frac32}\sqrt{\varepsilon}$$

(1), (2), (3) postaju: $\langle N\rangle=2\pi V\left(\frac{2m}{h^2}\right)^{\frac32}\int_0^\infty\frac{\varepsilon^{\frac12}\lambda e^{-\beta\varepsilon}}{1\pm\lambda e^{-\beta\varepsilon}}d\varepsilon \quad (4)$

$$\langle E\rangle=2\pi V\left(\frac{2m}{h^2}\right)^{\frac32}\int_0^\infty\frac{\varepsilon^{\frac32}\lambda e^{-\beta\varepsilon}}{1\pm\lambda e^{-\beta\varepsilon}}d\varepsilon \quad (5)$$

$$p=\pm 2\pi kT\left(\frac{2m}{h^2}\right)^{\frac32}\int_0^\infty\varepsilon^{\frac12}\ln(1\pm\lambda e^{-\beta\varepsilon})\,d\varepsilon \quad (6)$$

$$(4) \to \langle N\rangle = 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \int_0^\infty \varepsilon^{\frac{1}{2}} \lambda e^{-\beta\varepsilon} \sum_{n=1}^{\infty} (\mp 1)^{n-1} (\lambda e^{-\beta\varepsilon})^{n-1} d\varepsilon =$$

$$= 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \lambda^n \int_0^\infty \varepsilon^{\frac{1}{2}} e^{-\beta n\varepsilon} d\varepsilon = \begin{cases} \text{SMENA:} \\ \beta n\varepsilon = x \Rightarrow \varepsilon = \frac{x}{n\beta} \\ d\varepsilon = \frac{dx}{n\beta} \end{cases}$$

$$= 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \lambda^n \int_0^\infty \left(\frac{x}{n\beta}\right)^{\frac{1}{2}} e^{-x} \frac{dx}{n\beta} =$$

$$= 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \frac{\lambda^n}{n^{\frac{3}{2}}} (kT)^{\frac{3}{2}} \underbrace{\int_0^\infty x^{\frac{1}{2}} e^{-x} dx}_{\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}}$$

$$= \cancel{2}\pi V \frac{\sqrt{\pi}}{\cancel{2}} \left(\frac{2mkT}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \frac{\lambda^n}{n^{\frac{3}{2}}}$$

$\boxed{\lambda_T = \frac{h}{\sqrt{2\pi mkT}}}$ → DE - BROLJEVA TOPLOTNA TALASNA DUŽINA $\Rightarrow \frac{1}{\lambda_T^3} = \frac{(2\pi mkT)^{\frac{3}{2}}}{h^3} = \left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}}$

$$\boxed{\langle N\rangle = \frac{V}{\lambda_T^3} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \frac{\lambda^n}{n^{\frac{3}{2}}}} \quad (7)$$

$$(5) \to \langle E\rangle = 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \int_0^\infty \varepsilon^{\frac{3}{2}} \lambda e^{-\beta\varepsilon} \sum_{n=1}^{\infty} (\mp 1)^{n-1} (\lambda e^{-\beta\varepsilon})^{n-1} d\varepsilon =$$

$$= 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \lambda^n \int_0^\infty \varepsilon^{\frac{3}{2}} e^{-\beta n\varepsilon} d\varepsilon = \begin{cases} \text{SMENA:} \\ x = \beta n\varepsilon \Rightarrow \varepsilon = \frac{x}{\beta n} \Rightarrow \varepsilon^{\frac{3}{2}} = \left(\frac{x}{\beta n}\right)^{\frac{3}{2}} \\ d\varepsilon = \frac{dx}{\beta n} \end{cases}$$

$$= 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \lambda^n \int_0^\infty \left(\frac{x}{\beta n}\right)^{\frac{3}{2}} e^{-x} \frac{dx}{\beta n} =$$

$$= 2\pi V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} (kT)^{\frac{5}{2}} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \frac{\lambda^n}{n^{\frac{5}{2}}} \underbrace{\int_0^\infty x^{\frac{3}{2}} e^{-x} dx}_{\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi} = \frac{3\sqrt{\pi}}{4}}$$

$$= \frac{3}{2} \pi^{\frac{3}{2}} V \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} (kT)^{\frac{3}{2}} \cdot kT \sum_{n=1}^{\infty} (\mp 1)^{n-1} \frac{\lambda^n}{n^{\frac{5}{2}}} = \frac{3}{2} V \left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}} \cdot kT \sum_{n=1}^{\infty} (\mp 1)^{n-1} \frac{\lambda^n}{n^{\frac{5}{2}}}$$

$$\boxed{\langle E\rangle = \frac{3}{2} kT \frac{V}{\lambda_T^3} \sum_{n=1}^{\infty} (\mp 1)^{\substack{n+1 \\ 2n+1}} \frac{\lambda^n}{n^{\frac{5}{2}}}} \quad (8)$$

$$(6) \to p = \pm kT\, 2\pi \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \int_0^\infty \varepsilon^{\frac{1}{2}} \ln(1 \pm \lambda e^{-\beta\varepsilon}) d\varepsilon =$$

$$= \pm kT\, 2\pi \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \int_0^\infty \varepsilon^{\frac{1}{2}} \cdot \sum_{n=1}^{\infty} (-1)^{\substack{n+1 \\ 2n+1}} \frac{\lambda^n e^{-\beta n\varepsilon}}{n} d\varepsilon =$$

$$= \pm kT\, 2\pi \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (-1)^{\substack{n+1 \\ 2n+1}} \frac{\lambda^n}{n} \int_0^\infty \varepsilon^{\frac{1}{2}} e^{-\beta n\varepsilon} d\varepsilon = \begin{cases} \text{SMENA:} \\ x = \beta n\varepsilon \Rightarrow \varepsilon = \frac{x}{\beta n} \Rightarrow d\varepsilon = \frac{dx}{\beta n} \quad \varepsilon^{\frac{1}{2}} = \left(\frac{x}{\beta n}\right)^{\frac{1}{2}} \end{cases}$$

$$p = \pm 2\pi kT \left(\frac{2m}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (-1)^{\frac{n+1}{2n+1}} \frac{\lambda^n}{n} \int_0^{\infty} \frac{x^{\frac{1}{2}}}{(\beta n)^{\frac{1}{2}}} e^{-x} \frac{dx}{\beta n} =$$

$$= \pm \cancel{2}\pi kT \left(\frac{2mkT}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (-1)^{\frac{n+1}{2n+1}} \frac{\lambda^n}{n^{\frac{5}{2}}} \underbrace{\int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx}_{\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{\cancel{2}}}$$

$$= \pm kT \left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}} \sum_{n=1}^{\infty} (-1)^{\frac{n+1}{2n+1}} \frac{\lambda^n}{n^{\frac{5}{2}}}$$

$$\boxed{p = \pm kT \frac{1}{\lambda_T^3} \sum_{n=1}^{\infty} (-1)^{\frac{n+1}{2n+1}} \frac{\lambda^n}{n^{\frac{5}{2}}}} \quad (9)$$

Iz (7) $\rightarrow$ $\dfrac{\langle N \rangle}{V} = \rho = \dfrac{1}{\lambda_T^3} \displaystyle\sum_{n=1}^{\infty} (\mp 1)^{n-1} \frac{\lambda^n}{n^{\frac{3}{2}}}$

$$\rho = \frac{1}{\lambda_T^3}\left(\lambda \mp \frac{\lambda^2}{2^{\frac{3}{2}}} + \frac{\lambda^3}{3^{\frac{3}{2}}} + \dots\right)$$

$\hookrightarrow$ ALI NAMA TREBA $\lambda = a_1\rho + a_2\rho^2 + a_3\rho^3 + \dots$

$$\rho = \frac{1}{\lambda_T^3}\left[(a_1\rho + a_2\rho^2 + \dots) \mp \frac{1}{2^{\frac{3}{2}}}(a_1\rho + a_2\rho^2 + \dots)^2 + \dots\right] =$$

$$= \frac{1}{\lambda_T^3}\left[a_1\rho + a_2\rho^2 + \dots \mp \frac{1}{2^{\frac{3}{2}}} a_1^2\rho^2 + \dots\right] = \frac{1}{\lambda_T^3}\left[a_1\rho + \left(a_2 \mp \frac{1}{2^{\frac{3}{2}}} a_1^2\right)\rho^2 + \dots\right]$$

$\hookrightarrow$ $\lambda_T^3\rho + 0\cdot\rho^2 + 0\cdot\rho^3 + \dots = a_1\rho + \left(a_2 \mp \dfrac{1}{2^{\frac{3}{2}}} a_1^2\right)\rho^2 + \dots$

$$\left.\begin{aligned} & a_1 = \lambda_T^3 \\ & a_2 \mp \frac{1}{2^{\frac{3}{2}}} a_1^2 = 0 \Rightarrow a_2 = \pm \frac{a_1^2}{2^{\frac{3}{2}}} \Rightarrow a_2 = \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}} \end{aligned}\right\} \Rightarrow \boxed{\lambda = \lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots} \quad (10)$$

(10) $\rightarrow$ (8)

$$\langle E \rangle = \frac{3}{2}kT\frac{V}{\lambda_T^3}\sum_{n=1}^{\infty}(\mp 1)^{n-1} \frac{\left(\lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots\right)^n}{n^{\frac{5}{2}}} =$$

$$= \frac{3}{2}kT\frac{V}{\lambda_T^3}\left[\underbrace{\lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots}_{\text{ZA } n=1} \mp \underbrace{\frac{1}{2^{\frac{5}{2}}}\left(\lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots\right)^2}_{\text{ZA } n=2} + \dots\right]$$

$$= \frac{3}{2}kT\frac{V}{\lambda_T^3}\left[\lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots \mp \frac{\lambda_T^6}{2^{\frac{5}{2}}}\rho^2 + \dots\right] =$$

$$= \frac{3}{2}kTV\left[\rho + \left(\pm\frac{\lambda_T^3}{2^{\frac{3}{2}}} \mp \frac{\lambda_T^3}{2^{\frac{5}{2}}}\right)\rho^2 + \dots\right] = \frac{3}{2}kTV\left[\rho \pm \frac{\lambda_T^3}{2^{\frac{5}{2}}}\rho^2 + \dots\right]$$

$\hookrightarrow$ $\pm\dfrac{1}{2^{\frac{3}{2}}} \mp \dfrac{1}{2^{\frac{5}{2}}} = \dfrac{\pm 1}{2^{\frac{5}{2}}}$

$$\langle E\rangle = \frac{3}{2}kT \underbrace{V\rho}_{\langle N\rangle}\left[1 \pm \frac{\lambda_T^3}{2^{\frac{5}{2}}}\rho + \dots\right] \Rightarrow \boxed{\langle E\rangle = \frac{3}{2}\langle N\rangle kT\left[1 \pm \frac{\lambda_T^3}{2^{\frac{5}{2}}}\rho + \dots\right]}$$

$(10) \rightarrow (9)$

$$\frac{p}{kT} = \pm\frac{1}{\lambda_T^3}\sum_{n=1}^{\infty}(-1)^{\substack{n+1\\2n+1}}\frac{\left(\lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots\right)^n}{n^{\frac{5}{2}}} =$$

$$= \pm\frac{1}{\lambda_T^3}\Big[\underbrace{\pm\left(\lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots\right)}_{\text{ZA } n=1} \underbrace{\mp\frac{1}{2^{\frac{5}{2}}}\left(\lambda_T^3\rho \pm \frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots\right)^2}_{\text{ZA } n=2} + \dots\Big]$$

$$= \pm\frac{1}{\lambda_T^3}\left[\pm\lambda_T^3\rho \substack{+\\+}\frac{\lambda_T^6}{2^{\frac{3}{2}}}\rho^2 + \dots \mp \frac{\lambda_T^6}{2^{\frac{5}{2}}}\rho^2 + \dots\right] =$$

$$= \pm\frac{1}{\lambda_T^3}\left[\pm\lambda_T^3\rho + \left(\substack{+\\+}\frac{\lambda_T^6}{2^{\frac{3}{2}}} \mp \frac{\lambda_T^6}{2^{\frac{5}{2}}}\right)\rho^2 + \dots\right] = \pm\left[\pm\rho + \frac{\lambda_T^3}{2^{\frac{5}{2}}}\rho^2 + \dots\right]$$

$$\hookrightarrow \substack{+\\+}\frac{1}{2^{\frac{3}{2}}} \mp \frac{1}{2^{\frac{5}{2}}} \Rightarrow \frac{\pm 2 \mp 1}{2^{\frac{5}{2}}} = \frac{1}{2^{\frac{5}{2}}}$$

$$\frac{p}{kT} = \rho\left[\substack{+\\+}1 \pm \frac{\lambda_T^3}{2^{\frac{5}{2}}}\rho + \dots\right] \Rightarrow \boxed{p = \rho kT\left[1 \pm \frac{\lambda_T^3}{2^{\frac{5}{2}}}\rho + \dots\right]}$$

$\rightarrow$ SLUČAJ BOZONA BEZ MASE:

$$g(\mathcal{E}) = 4\pi V\frac{\mathcal{E}^2}{(hc)^3}$$

(1), (2), (3) postaju: $\langle N\rangle = \frac{4\pi V}{(hc)^3}\int_0^\infty \frac{\mathcal{E}^2\lambda e^{-\beta\mathcal{E}}}{1-\lambda e^{-\beta\mathcal{E}}}d\mathcal{E}$ (11)

$$\langle E\rangle = \frac{4\pi V}{(hc)^3}\int_0^\infty \frac{\mathcal{E}^3\lambda e^{-\beta\mathcal{E}}}{1-\lambda e^{-\beta\mathcal{E}}}d\mathcal{E} \quad (12)$$

$$p = -\frac{4\pi kT}{(hc)^3}\int_0^\infty \mathcal{E}^2\ln(1-\lambda e^{-\beta\mathcal{E}})d\mathcal{E} \quad (13)$$

RAZVOJI KOJE ĆEMO KORISTITI:

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} x^{n-1}$$

$$\ln(1-x) = \sum_{n=1}^{\infty} -\frac{x^n}{n}$$

$$(11) \rightarrow \langle N\rangle = \frac{4\pi V}{(hc)^3}\int_0^\infty \mathcal{E}^2\lambda e^{-\beta\mathcal{E}}\sum_{n=1}^{\infty}(\lambda e^{-\beta\mathcal{E}})^{n-1}d\mathcal{E} =$$

$$= \frac{4\pi V}{(hc)^3}\sum_{n=1}^{\infty}\lambda^n\int_0^\infty \mathcal{E}^2\cdot e^{-\beta n\mathcal{E}}d\mathcal{E} = \left\{\begin{array}{l}\text{SMENA:}\\ \beta n\mathcal{E} = x \Rightarrow \mathcal{E}^2 = \frac{x^2}{(\beta n)^2} \Rightarrow d\mathcal{E} = \frac{dx}{\beta n}\end{array}\right.$$

$$= \frac{4\pi V}{(hc)^3}\sum_{n=1}^{\infty}\lambda^n\int_0^\infty \frac{x^2}{(\beta n)^2}e^{-x}\frac{dx}{\beta n} = 4\pi V\left(\frac{kT}{hc}\right)^3\sum_{n=1}^{\infty}\frac{\lambda^n}{n^3}\underbrace{\int_0^\infty x^2e^{-x}dx}_{\Gamma(3)=2!=2}$$

$$\langle N\rangle = 8\pi V\left(\frac{kT}{hc}\right)^3\sum_{n=1}^{\infty}\frac{\lambda^n}{n^3} \Rightarrow \rho = \frac{\langle N\rangle}{V} = 8\pi\left(\frac{kT}{hc}\right)^3\sum_{n=1}^{\infty}\frac{\lambda^n}{n^3}$$

$$\boxed{\rho = \frac{\langle N\rangle}{V} = \pi\left(\frac{2kT}{hc}\right)^3\sum_{n=1}^{\infty}\frac{\lambda^n}{n^3}} \quad (14)$$

$$(12) \to \langle E\rangle = \frac{4\pi V}{(hc)^3}\int_0^{\infty}\varepsilon^3\lambda e^{-\beta\varepsilon}\sum_{n=1}^{\infty}(\lambda e^{-\beta\varepsilon})^{n-1}d\varepsilon$$

$$\boxed{\langle E\rangle = 3kT\left(\frac{2kT}{hc}\right)^3\pi V\sum_{n=1}^{\infty}\frac{\lambda^n}{n^4}} \quad (15)$$

$$(13) \to p = -\frac{4\pi kT}{(hc)^3}\int_0^{\infty}\varepsilon^2\sum_{n=1}^{\infty} - \frac{(\lambda e^{-\beta\varepsilon})^n}{n}d\varepsilon =$$

$$= +\frac{4\pi kT}{(hc)^3}\sum_{n=1}^{\infty}\frac{\lambda^n}{n}\int_0^{\infty}\varepsilon^2\cdot e^{-\beta n\varepsilon}d\varepsilon = \left\{\begin{array}{l}\text{SMENA:}\\ \beta n\varepsilon = x \Rightarrow \varepsilon^2 = \frac{x^2}{(\beta n)^2} \Rightarrow d\varepsilon = \frac{dx}{\beta n}\end{array}\right.$$

$$= \frac{4\pi kT}{(hc)^3}\sum_{n=1}^{\infty}\frac{\lambda^n}{n}\int_0^{\infty}\frac{x^2}{(\beta n)^2}e^{-x}\frac{dx}{\beta n} = 4\pi kT\left(\frac{kT}{hc}\right)^3\sum_{n=1}^{\infty}\frac{\lambda^n}{n^4}\underbrace{\int_0^{\infty}x^2e^{-x}dx}_{\Gamma(3)=2!=2}$$

$$\boxed{p = kT\left(\frac{2kT}{hc}\right)^3\pi\sum_{n=1}^{\infty}\frac{\lambda^n}{n^4}} \quad (16)$$

$$(14) \to \rho = \pi\left(\frac{2kT}{hc}\right)^3\left[\lambda + \frac{1}{8}\lambda^2 + \frac{1}{27}\lambda^3 + \ldots\right]$$

NAMA TREBA $\lambda = a_1\rho + a_2\rho^2 + \ldots$

$$\rho = \pi\left(\frac{2kT}{hc}\right)^3\left[a_1\rho + a_2\rho^2 + \ldots + \frac{1}{8}(a_1\rho + a_2\rho^2 + \ldots)^2 + \ldots\right] =$$

$$= \pi\left(\frac{2kT}{hc}\right)^3\left[a_1\rho + a_2\rho^2 + \ldots + \frac{1}{8}a_1^2\rho^2 + \ldots\right] =$$

$$= \pi\left(\frac{2kT}{hc}\right)^3\left[a_1\rho + \left(a_2 + \frac{1}{8}a_1^2\right)\rho^2 + \ldots\right]$$

$$\hookrightarrow \frac{1}{\pi}\left(\frac{hc}{2kT}\right)^3\rho + 0\cdot\rho^2 + \ldots = a_1\rho + \left(a_2 + \frac{1}{8}a_1^2\right)\rho^2 + \ldots$$

$$\left.\begin{array}{l} a_1 = \frac{1}{\pi}\left(\frac{hc}{2kT}\right)^3 \\ a_2 + \frac{1}{8}a_1^2 = 0 \Rightarrow a_2 = -\frac{1}{8\pi^2}\left(\frac{hc}{2kT}\right)^6 \end{array}\right\} \Rightarrow \boxed{\lambda = \frac{1}{\pi}\left(\frac{hc}{2kT}\right)^3\rho - \frac{1}{8\pi^2}\left(\frac{hc}{2kT}\right)^6\rho^2 + \ldots} \quad (17)$$

$(17) \to (15)$

$$\langle E \rangle = 3kT\left(\frac{2kT}{hc}\right)^3 \pi V\left[\underbrace{\frac{1}{\pi}\left(\frac{hc}{2kT}\right)^3 \rho - \frac{1}{8\pi^2}\left(\frac{hc}{2kT}\right)^6 \rho^2 + \ldots}_{ZA \quad n=1} + \underbrace{\frac{1}{2^4}\cdot\frac{1}{\pi^2}\left(\frac{hc}{2kT}\right)^6 \rho^2 + \ldots}_{ZA \quad n=2}\right]$$

$$\langle E \rangle = 3VkT\left[\rho - \frac{1}{8\pi}\left(\frac{hc}{2kT}\right)^3 \rho^2 + \ldots + \frac{1}{16\pi}\left(\frac{hc}{2kT}\right)^3 \rho^2 + \ldots\right] =$$

$$= 3\overset{\langle N \rangle}{V\rho} kT\left[1 + \left(-\frac{1}{8\pi}\left(\frac{hc}{2kT}\right)^3 + \frac{1}{16\pi}\left(\frac{hc}{2kT}\right)^3\right)\rho + \ldots\right]$$

$$\boxed{\langle E \rangle = 3\langle N \rangle kT\left[1 - \frac{1}{16\pi}\left(\frac{hc}{2kT}\right)^3 \rho + \ldots\right]}$$

$(17) \to (16)$

$$p = kT\left(\frac{2kT}{hc}\right)^3 \pi\left[\frac{1}{\pi}\left(\frac{hc}{2kT}\right)^3 \rho - \frac{1}{8\pi^2}\left(\frac{hc}{2kT}\right)^6 \rho^2 + \ldots + \frac{1}{16\pi^2}\left(\frac{hc}{2kT}\right)^6 \rho^2 + \ldots\right]$$

$$= \rho kT\left(\frac{2kT}{hc}\right)^3 \pi\left[\frac{1}{\pi}\left(\frac{hc}{2kT}\right)^3 + \left(-\frac{1}{8\pi^2}\left(\frac{hc}{2kT}\right)^6 + \frac{1}{16\pi^2}\left(\frac{hc}{2kT}\right)^6\right)\rho + \ldots\right]$$

$$= \rho kT\left[1 + \left(-\frac{1}{8\pi}\left(\frac{hc}{2kT}\right)^3 + \frac{1}{16\pi}\left(\frac{hc}{2kT}\right)^3\right)\rho + \ldots\right] =$$

$$= \rho kT\left[1 - \frac{1}{16\pi}\left(\frac{hc}{2kT}\right)^3 \rho + \ldots\right]$$

$$\boxed{p = \frac{\langle N \rangle kT}{V}\left[1 - \frac{1}{16\pi}\left(\frac{hc}{2kT}\right)^3 \rho + \ldots\right]}$$